December 4, 1996 WOCOMAL VARSITY MEET

ROUND I: Arithmetic - Percent, interest, discount, fractions, and decimals

ALL ANSWERS MUST BE IN THE FORM SPECIFIED IN THE PROBLEM

1. The wholesale price of a sweater is $20. If the sweater is marked up 67% and then discounted 15%, what will be the selling price of the sweater to the nearest cent?

2. Express 20% of $\dfrac{\frac{4}{7} - \frac{1}{14}}{\frac{189}{28} - 0.25}$ as a reduced fraction.

3. One can rent a couch for a few dollars a week, and with a small payment at the end, own the couch. L. Euler pays a service fee of $25 and rent of $23.69 per month for 30 months and an end payment of $50. If the couch could have been bought for $239, what percent of the $239 does Ms. Euler actually pay? (Nearest whole %)

ANSWERS

(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Algonquin, Bancroft, Burncoat, Holy Name

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ROUND II: Algebra I - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If the area of a square of side p is 7, what is the area of a square with side $2p$?

2. A crew of thirty people did one-fourth of a job in eight days. Then it became necessary to complete the job in the next five days. If all worked at the same rate, how many additional workers were added to the crew of thirty to accomplish this task?

3. If $p, q \in \{ -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \}$,

   find all ordered pairs (p, q) such that $\frac{p+q}{p} = \frac{2}{q}$

ANSWERS

(1 pt.) 1._____

(2 pts) 2._____

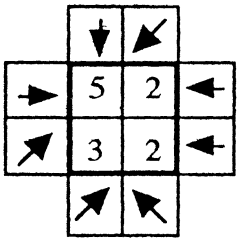
(3 pts) 3._____

Auburn, Tantasqua, Worcester Academy

ROUND III: Problem Solving

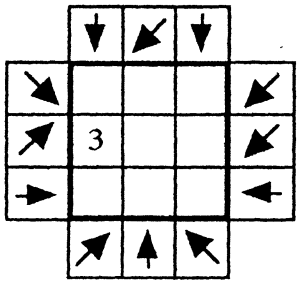
SPECIAL NOTE: Problem 1 is worth 2 points, problem 2a is worth 1 point, and problem 2b is worth 3 points. Your answers must be in the answer section.

1. Bunny picked four different numbers between 0 and 36. The average of the first two numbers is 3. The sum of the second and third is 25. The average of the third and fourth is 25.5. The average of the second, third, and fourth is $18\ 2/3$. What four numbers did she pick?

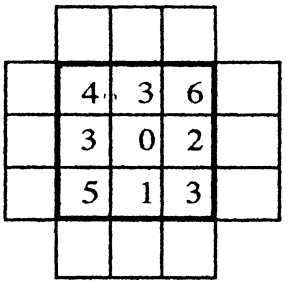
2. 

The arrows point toward one or more numbers in the inner squares Each number tells how many arrows point toward it. Exactly eight directions are allowed for the arrows, horizontal, vertical, and 45° rotations, and each arrow must point toward at least one numbered square. Based on this:

a) Fill in the other eight numbers

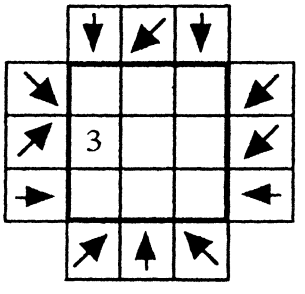


b) Fill in twelve arrows

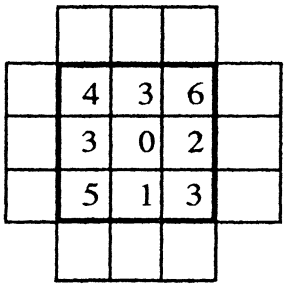


ANSWER
(2 pts) 1._____

(1 pt.) 2a.



(3 pts) 2b.



Bartlett, Mass Academy

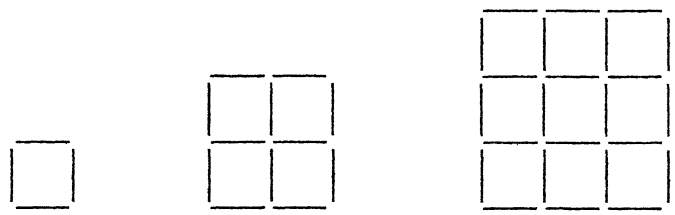
December 4, 1996 WOCOMAL VARSITY MEET

ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The fourth term of a sequence is 4 and the sixth term is 6. Every term after the second is the sum of the two preceding terms. Find the eighth term.

2. Find the sum of all the positive even integers with three digits.

3. 

4 Straws 12 Straws 24 Straws

Write a formula for the number of straws needed for a square pattern of side n, where n is the number of straws in a side.

ANSWERS

(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, Southbridge, Tahanto

December 4, 1996 WOCOMAL VARSITY MEET

ROUND V: Matrix and determinant operations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for c: $\begin{vmatrix} 6c & 3 \\ c & 7 \end{vmatrix} = 13$

2. Find $(A - B)^2 + C$ for $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$

3. Find X^T, the transpose of Matrix X, given $\begin{bmatrix} -6 & -3 \\ 3 & 1 \end{bmatrix} X = \begin{bmatrix} 9 & 12 & 0 \\ -4 & 5 & -2 \end{bmatrix}$

ANSWERS

(1 pt.) 1._____

(2 pts) 2. $\begin{bmatrix} \quad & \quad \end{bmatrix}$

(3 pts) 3. $\begin{bmatrix} \quad & \quad \end{bmatrix}$

Bartlett, Clinton, St. John's

December 4, 1996 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET (2 points each)

1. Express the product $(1+\frac{3}{4})(1+\frac{3}{5})(1+\frac{3}{6})(1+\frac{3}{7})\ldots(1+\frac{3}{21})(1+\frac{3}{22})$ as a rational number.

2. Let x denote the width of a rectangle with area 25. Express its perimeter in terms of x.

3. An organization of 105 people set up a phone chain so that the initial contact person calls two people, each of whom calls two others, and so on until all have been contacted. What is the maximum number of people who do not need to make a call?

4. The formula for the sum of the first n terms of a sequence is $S_n = \frac{n(3n-1)}{2}$ Find the 20th term.

5. Solve for x: $\begin{vmatrix} 2 & 4 & 12 \\ x & x & 4 \\ 0 & x & 2 \end{vmatrix} = 45$

6. How many times do the hour and minute hands of a clock cross in 12 hours?

7. Solve for x. $|x|^2 = 2|x|+5$. If radicals occur, do not approximate, but give exact simple radical form.

8. If f(x) equals the square of the opposite of x, find all values of x for which

$$f(x) = \frac{f(-2)+f(-1)+f(0)+f(1)}{f(\sqrt{6})}$$

9. Find the equation in $y = mx + b$ form for a line passing no farther than one unit vertically away from each of these five points: (0,3), (1,1), (2,0), (3,1), and (4,1). If there is no such line, say so.

Bromfield, Hudson, Mass Academy, Quaboag, St. John's, Tantasqua, Westboro, Worcester Academy

Round 1

1. $SP = \$\ 20 \times 1.67 \times 0.85 = \$\ 28.39$

2. Using a calculator with a fraction key, $\frac{1}{5} \times (\frac{4}{7} - \frac{1}{14}) \div (\frac{189}{28} - \frac{1}{4}) = \frac{1}{65}$.

3. Normally, the service fee and end payment are one-time payments. So, under this plan the couch would cost $25 + 50 + 30 \times 23.69 = 785.70$. This is $\frac{785.7}{239} = 3.2874 = 329\%$.

Round 2

1. If $p^2 = 7$, then $(2p)^2 = 4p^2 = 28$.

2. Since $30 \times r \times 8 = \frac{1}{4}$, the rate at which one person works is $r = \frac{1}{960}$ th of the job per day. Let w be the number of additional workers required for the last 3/4 of the job. Then, $(30 + w) \times \frac{1}{960} \times 5 = \frac{3}{4}$ and $w = 114$.

3. Since $pq + q^2 = 2p$, it is easiest to solve for $p = \frac{q^2}{2 - q}$. It doesn't take long to substitute the eleven values of q and determine that (1,-2), (1,1), and (0,0) work.

Round 3

This question again raises the troubling prospect that the kid holding the calculator with SIMULT has an unfair advantage. The four equations: $\frac{w+x}{2} = 3$, $x + y = 25$, $\frac{y+z}{2} = 25.5$, *and* $\frac{x+y+z}{3} = 18\frac{2}{3}$ become w + x = 3 , x + y = 25 , y + z = 51 , and x + y + z = 56 . So on the calculator enter these: 1,1,0,0,3 0,1,1,0,25 0,0,1 1,51 0,1,1,1,56 to get (1,5,20,31) too easily. Either these questions have to go or those calculators have to go.

2. This is an interesting problem. The first part is simple counting and gently introduces the student to the process. The second part, however, requires some experimentation with pencil and eraser. You simply have to push the pencil on paper and do it, and you hope you can do it in whatever time you have remaining.

Round 4

1. We start with the fourth term. Suppose the fifth term is x . Then 4 + x = 6 and x = 2. From our start, the terms are 4 , 2 , 6 , 8 , **14** , ...

2. ie. Find 100 + 102 + 104 + ... + 998 . Arithmetic, d=2, t_1=100, t_n=998 . Thus, 100 + (*n*-1)2 = 998 $\Rightarrow$ *n* = 450 . So, $S_{450} = \frac{450}{2}(100 + 998) = 247{,}050$. But if you have one of those fancy calculators, then sum seq(x,x,100,998,2) = 247,050 .

3. In each direction the number of straws is n(n+1). Therefore, ans = 2n(n+1) .

Round 5

1. $42c - 3c = 13 \Rightarrow c = \frac{1}{3}$

2. $A - B = \begin{bmatrix} 1 & 5 \\ 1 & -1 \end{bmatrix}$, $(A - B)^2 = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$. So, $(A - B)^2 + C = \begin{bmatrix} 8 & 3 \\ 3 & 8 \end{bmatrix}$.

3. $\begin{bmatrix} -6 & -3 \\ 3 & 1 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 3 \\ -3 & -6 \end{bmatrix}$. So, $X = \frac{1}{3}\begin{bmatrix} 1 & 3 \\ -3 & -6 \end{bmatrix} \cdot \begin{bmatrix} 9 & 12 & 0 \\ -4 & 5 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 9 & -2 \\ -1 & -22 & 4 \end{bmatrix}$.
But then it must be transposed.

Team

1. The pattern $\left(\frac{7}{4}\right)\left(\frac{8}{5}\right)\left(\frac{9}{6}\right)\left(\frac{10}{7}\right) \ldots \left(\frac{24}{21}\right)\left(\frac{25}{22}\right)$ collapses to $\frac{23 \times 24 \times 25}{4 \times 5 \times 6} = 115$.

2. $y = \frac{25}{x}$. So, $p = 2x + 2y = 2x + \frac{50}{x}$.

3. The first contacted $1 + 2 + 4 + 8 + 16 + 32 = 63$ employees could in turn make a total of 126 calls. This overshoots the mark by $21 + 1$(first called) $= 22$ which requires 11 callers This means that the number of callers minimally needs to be $63 - 11 = 52$.
So, $105 - 52 = 53$ do not need to make a call.

4. $t_{20} = S_{20} - S_{19} = \frac{20 \times 59}{2} - \frac{19 \times 56}{2} = 58$.

5. $4x^2 + 0 + 12x^2 - (8x + 8x + 0) = 45 \quad \Rightarrow \quad x = -\frac{3}{2}$ *or* $+\frac{5}{2}$.

6. They start together and "cross" each hour except between 0 and 1 and between 11 and 12. Answer is thus $10 + 1 = 11$ times.

7. $A^2 - 2A - 5 = 0$ has solutions of $1 \pm \sqrt{6}$. But $1 - \sqrt{6} < 0$. So, $|x| = 1 + \sqrt{6}$ only. And this means $x = \pm(1 + \sqrt{6})$.

8. $f(x) = (-x)^2 = x^2$. So, $x^2 = \frac{4 + 1 + 0 + 1}{6} = 1$ and $x = \pm 1$.

9. After plotting the points, notice that the line thru (0,2) and (4,0) is peachy-keen.
It's equation is $\frac{x}{4} + \frac{y}{2} = 1$ *or* $y = -\frac{1}{2}x + 2$.